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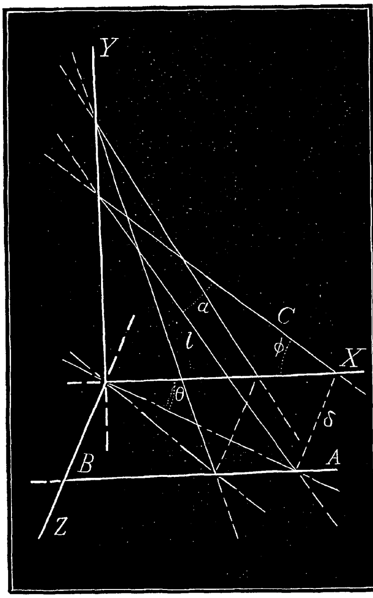
a given time such values as make the orbits decidedly elliptic, and permit to them the vibrations caused by the action of the members of the system, without inteference or in other words intersections. There can hardly be a doubt that our solar system as composed of the sun and eight principal planets fulfills these conditions.

* * *

SOLUTION OF A PROBLEM.

BY E. W. HYDE, PROF. OF MATH. IN PA. MILITARY ACADEMY, CHESTER, PA.

Locus of a right line moving so that two fixed points of it remain constantly in two other right lines, which are perpendicular to each other but do not intersect.



Let the axis of y and line $A\ B$ be the directrices.

A line in $X\ Y$ through two points is

$$\frac{y-y_1}{x-x_1} = \frac{y_1-y_2}{x_1-x_2}, \quad \ldots\ldots (a)$$

or, $(y-y_1)(x_1-x_2)=(x-x_1)(y_1-y_2)$.

For the line c we have

$$x_1 = 0,\ y_1 = c \sin \psi,$$
$$x_2 = c \cos \psi,\ y_2 = 0.$$

$$\text{But } \frac{\delta}{c \cos \psi} = \tan \theta = \frac{z}{x}.$$

$$\therefore \cos \psi = \frac{\delta x}{c z}.$$

$$\therefore \sin \psi = \sqrt{1-\cos^2 \psi} = \sqrt{1-\frac{\delta^2 x^2}{c^2 z^2}}$$

Substituting these values in equation (a)

$$\left(y - x\sqrt{1-\frac{\delta^2\ {}^2}{c^2 z^2}}\right)\left(-\frac{\delta x}{z}\right) = x\left(c\sqrt{1-\frac{\delta^2 x^2}{c^2 z^2}}\right),$$

$$\text{or } c\,x\left(\frac{\delta}{z}-1\right)\sqrt{1-\frac{\delta^2 x^2}{c^2 z^2}} = \frac{\delta\,x\,y}{z},$$

and by squaring and reducing, we obtain,

$$c^2(\delta z - z^2)^2 - \delta^3 x^2 (\delta - 2z) - \delta^2 z^2 (x^2 + y^2) = 0 \ldots\ldots. (b)$$

the equation of the surface. If we intersect the surface by a plane $y = t x$ we have in the equation only the square of x, hence the curve of section is symmetrical about z, and this is the axis of the surface. If we move the origin to a point on z half way between the two directrices we obtain a more symmetrical form of the equation.

Let $z' = z - \frac{\delta}{2}$ or $z = z' + \frac{\delta}{2}$, and in the result place $\frac{\delta}{2} = a$, then after reduction, and dropping primes,

$$z^4 - \frac{4a^2}{c^2} z^2 (x^2+y^2) + \frac{8a^3}{c^2} z (x^2-y^2) - 2a^2 \left\{ (x^2+y^2) \frac{2a^2}{c^2} + z^2 \right\} = -a^4. \ldots\ldots.. (c)$$

Intersect by the plane $z = n a$, and we have after reduction

$$\frac{4}{c^2}(n-1)^2 x^2 + \frac{4}{c^2}(n+1)^2 y^2 = (n^2-1)^2, \ldots\ldots\ldots. (d)$$

an ellipse whose semi-axes are

$$A = \frac{c(n+1)}{2} \text{ and } B = \frac{c(n-1)}{2}.$$

If $n = 0$, $A = B$, and the curve is a circle. If $n = \infty$, we have likewise,

$$A = B. \quad A - B, \text{ or } \frac{cn + c}{2} - \frac{cn - c}{2} = c,$$

hence there is a constant difference between the transverse and conjugate axes. When however, n becomes less than unity, c is the *sum* of the semi-axes instead of their difference. For when $n = 1$, $B = 0$, and the section is a right line, and when $n < 1$, B is *negative*, hence the *algebraic difference* is the *sum*. If $n = -1$, $A = 0$.

The orthographic projection of this surface on any plane parallel to the axis is an hyperbola.

To prove this we shall obtain the equation of the tangent cylinder whose elements are perpendicular to the axis.

The equation of a section perpendicular to z is (from (d) above)

$$\frac{x^2}{\frac{c^2(n+1)^2}{4}} + \frac{y^2}{\frac{c^2(n-1)^2}{4}} = 1.$$

A line tangent to an ellipse is $y - m\,x = \sqrt{m^2\,a^2 + b^2}$. But

$$a = \frac{c(n+1)}{2}, \text{ and } b = \frac{c(n-1)}{2},$$

$$\text{hence } y - m\,x = \sqrt{\frac{m^2\,c^2\,(n+1)^2}{4} + \frac{c^2\,(n-1)^2}{4}},$$

which after reduction becomes, since $n = \frac{z}{a}$,

$$4\,a^2\,m^2\,x^2 - 8\,a^2\,m\,x\,y + 4\,a^2\,y^2 - c^2\,(m^2+1)\,z^2$$
$$-2\,a\,c^2\,(m^2-1)\,z - a^2\,c^2\,(m^2+1) = 0. \;\ldots\ldots(e)$$

This is the tangent cylinder, and its sections are hyperbolas; as for instance that by the plane $y = 0$, for which we have

$$4\,a^2\,m^2\,x^2 - c^2\,(m^2+1)\,z^2 - 2\,a\,c^2\,(m^2-1)\,z - a^2\,c^2\,(m^2+1) = 0.$$

If $m = 0$ in equation (e) it reduces to $2\,a\,y \mp a\,c \pm c\,z = 0$,

$$\text{or } y = \mp\frac{c}{2a}\,z \pm \frac{c}{2},$$

two intersecting planes.

If $m = \infty$ we have similarly $2\,a\,x \pm c\,z \pm a\,c = 0$,

$$\text{or } x = \mp\frac{c}{2a}\,z \pm \frac{c}{2}.$$

Motion of a Sphere on an Inclined Plane.—By Walter Siverly, Oil City, Pa.—Let m = the mass of the sphere, a its radius, k its radius of gyration, β the inclination of the plane, u the coefficient of the dynamical friction between the plane and sphere, R the reaction of the plane on the sphere, s the space passed over by the center of the sphere in the time t, ϕ the angle through which it has revolved about its center, and let it be projected down the plain with the velocity v, and at the same time impressed with an angular velocity w. For its motion,

$$m\frac{d^2\,s}{d\,t^2} = m\,g\,\sin\beta - u\,R, \quad \ldots\ldots\ldots\ldots\ldots(1)$$

$$m\,k^2\,\frac{d^2\phi}{d\,t^2} = a\,u\,R, \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

But $R = m\,g\cos\beta$, hence from (1) and (2)

$$\frac{d^2\,s}{d\,t^2} = g\,(\sin\beta - u\cos\beta), \quad \ldots\ldots\ldots\ldots\ldots(3)$$

$$k^2\,\frac{d^2\phi}{d\,t^2} = a\,g\,u\cos\beta. \quad \ldots\ldots\ldots\ldots\ldots\ldots(4)$$